

Task 1. The numerator of a fraction is equal to 5. If we increase the denominator of that fraction by 27 and the numerator by 15, the value of the fraction does not change.

What is the value of the fraction?

Solution

Let x be the denominator of the fraction.

$$\frac{5}{x} = \frac{20}{x+27}$$

$$20x = 5(x + 27)$$

$$20x = 5x + 135$$

$$15x = 135$$

$$x = 9$$

$$\text{Answer: } \frac{5}{9}$$

Task 2. The larger excavator dug $\frac{1}{3}$ lengths of trench in 4 hours. The smaller excavator finished digging the trench in 10 hours. How long would the job have taken if both excavators had been working together at the same time?

Solution

$$\text{The efficiency of the larger excavator: } W_1 = \frac{\frac{1}{3}}{4} = \frac{1}{12} \left[\frac{\text{trench}}{\text{hour}} \right]$$

$$\text{The efficiency of the smaller excavator: } W_2 = \frac{\frac{2}{3}}{10} = \frac{1}{15} \left[\frac{\text{trench}}{\text{hour}} \right]$$

Let t be the time needed to finish the job if both excavators dug together.

$$t \cdot \left(\frac{1}{12} + \frac{1}{15} \right) = 1$$

$$t \cdot \left(\frac{27}{12 \cdot 15} \right) = 1$$

$$t = \frac{12 \cdot 15}{27} = \frac{20}{3} = 6 \frac{2}{3}$$

$$\text{Answer: } 6 \frac{2}{3} \text{ h}$$

Task 3. A car and a motorcycle set off on a 120 km journey at the same time. The car arrived 10 min later than the motorcycle. What were the average speeds of both vehicles if the motorcycle was travelling 8 km/h faster than the car?

Solution

	Distance [km]	Speed [km/h]	Time [h]
Car	120	v	t
Motorcycle	120	$v + 8$	$t - \frac{1}{6}$

$$(1) v \cdot t = 120 \rightarrow t = \frac{120}{v}$$

$$(2) (v + 8) \cdot \left(t - \frac{1}{6} \right) = 120$$

$$(3) (v + 8) \cdot \left(\frac{120}{v} - \frac{1}{6} \right) = 120$$

$$(3) 120 - \frac{v}{6} + \frac{8 \cdot 120}{v} - \frac{8}{6} = 120$$

$$(4) -\frac{v}{6} + \frac{8 \cdot 120}{v} - \frac{8}{6} = 0 \quad / \cdot (-6v)$$

$$(4) v^2 + 8v - 6 \cdot 8 \cdot 120 = 0$$

$$(5) \Delta = 64 \cdot 361 \quad v = 72$$

Answer: The average speed was 72 [km/h] and the average speed of the motorcycle was [80km/h]