Simple Rational Equations and Simple Absolute Value Equations – Solutions and Answers

Task 1.1. Solve the following equation algebraically.

$$\frac{x-1}{x+2} = \frac{x-3}{x-4}$$

Solution:

Domain $D = R - \{-2, 4\}$

$$\frac{x-1}{x+2} = \frac{x-3}{x-4}$$
$$(x-1)(x-4) = (x+2)(x-3)$$
$$x^2 - 4x - x + 4 = x^2 - 3x + 2x - 6$$
$$x^2 - 5x + 4 = x^2 - x - 6$$
$$-4x = -10$$
$$x = 2\frac{1}{2} \in D$$

Answer: $x = 2\frac{1}{2}$. The equation is consistent.

Task 1.2. Solve the following equation algebraically.

$$\frac{x^2 - 2x}{x} = x - 2$$

Solution:

Domain $D = R - \{0\}$

$$\frac{x^2 - 2x}{x} = \frac{x-2}{1}$$
$$x^2 - 2x = x^2 - 2x$$
$$0x = 0$$
$$x \in R \cap D$$
$$x \in R - \{0\}$$

Answer: Every real number in the domain satisfies the equation. This is an identity equation.

Task 1.3. Solve the following equation algebraically.

$$\frac{x-1}{x-2}=\frac{x-3}{x-4}$$

Solution:

Domain $D = R - \{2, 4\}$

$$\frac{x-1}{x-2}=\frac{x-3}{x-4}$$

$$(x-1)(x-4) = (x-2)(x-3)$$

$$x^2 - 4x - x + 4 = x^2 - 3x - 2x + 6$$

$$x^2 - 5x + 4 = x^2 - 5x + 6$$

$$0x = 2$$

Answer: No real number satisfies this equation. The equation is inconsistent.

Task 2.1. Solve the equation algebraically and graphically.

$|x-1|=3$

Algebraic solution

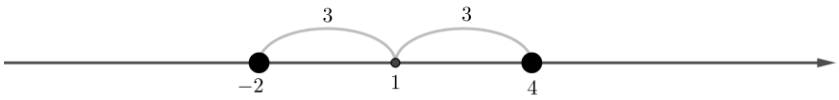
$x-1=3$ or $x-1=-3$

$x=4$ or $x=-2$

Answer: $x \in \{-2,4\}$

Graphical Solution

The distance x from 1 has to be equal 3.



Answer: $x \in \{-2,\ 4\}$

Task 2.2. Solve the equation algebraically and graphically.

$|x+2|=5$

Algebraic solution

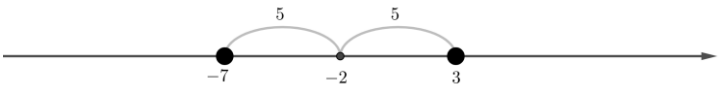
$x+2=5$ or $x+2=-5$

$x=3$ or $x=-7$

Answer: $x \in \{-7,\ 3\}$

Graphical Solution

The distance x from -2 has to be equal 5.



Answer: $x \in \{-7,\ 3\}$