Trigonometry of convex angles - Answers

Task 1. (5p) Calculate $\sin\alpha$ $\cos\alpha$, $\tan\alpha$, $\cot\alpha$.

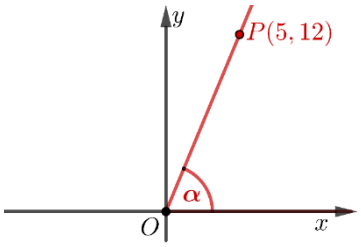
(a)

$\cos\alpha = \frac{5}{13}$

$\sin\alpha = \frac{12}{13}$

$\tan\alpha = \frac{12}{5}$

$\cot\alpha = \frac{5}{12}$



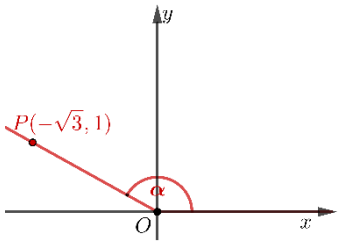
(b)

$\cos\alpha = \frac{-\sqrt{3}}{2}$

$\sin\alpha = \frac{1}{2}$

$\tan\alpha = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\cot\alpha = -\sqrt{3}$



Task 2. You are given an angle α such that $\alpha \in (90°, 180°)$ and $\sin\alpha = \frac{2}{3}$.

Use trigonometric identities to calculate $\cos\alpha$, $\tan\alpha$ and $\cot\alpha$

Solution

$\sin^2\alpha + \cos^2\alpha = 1$

$\cos^2\alpha = 1 - \sin^2\alpha$

$\cos^2\alpha = 1 - \left(\frac{2}{3}\right)^2$

$\cos^2\alpha = \frac{5}{9}$ / $\cos\alpha < 0$ because $\alpha \in (90°, 180°)$

$\cos\alpha = -\frac{\sqrt{5}}{3}$

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$

$\tan\alpha = \frac{\frac{2}{3}}{-\frac{\sqrt{5}}{3}} = -\frac{2}{3}\cdot\frac{3}{\sqrt{5}} = -\frac{2}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$

Answer: $\cos\alpha = -\frac{\sqrt{5}}{3}$ and $tan\alpha = -\frac{3\sqrt{5}}{5}$

Solution

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$

$\frac{\sin\alpha}{\cos\alpha} = 7$

$sin\alpha = 7\cdot cos\alpha$

$\sin^2\alpha + \cos^2\alpha = 1$

$49\cdot cos^2\alpha + cos^2\alpha = 1$

$50\cdot cos^2\alpha = 1$

$cos^2\alpha = \frac{1}{50}$ / $\cos\alpha < 0$ because $\alpha \in (90°, 180°)$

$\cos\alpha = -\frac{\sqrt{2}}{10}$

$\sin\alpha = 7\cdot\cos\alpha = -\frac{7\sqrt{2}}{10}$

Answer: $\cos\alpha = -\frac{\sqrt{2}}{10}$ $\sin\alpha = -\frac{7\sqrt{2}}{10}$

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Task 3. You are given an angle α such that $\alpha \in (0°, 180°)$ and $\tan\alpha = -7$.

Use trigonometric identities to calculate $\sin\alpha\ \cos\alpha$, $\cot\alpha$.

Solution

$\tan\alpha\ <\ 0$, so that $\alpha \in (90°, 180°)$

$\tan\alpha = \frac{sin\alpha}{cos\alpha}$

$$\frac{\sin\alpha}{\cos\alpha} = -7$$

$$\sin\alpha = -7 \cdot \cos\alpha$$

$\sin^2\alpha + \cos^2\alpha = 1$

$$49 \cdot \cos^2\alpha + \cos^2\alpha = 1$$

$$50 \cdot \cos^2\alpha = 1$$

$$\cos^2\alpha = \frac{1}{50}$$

$$\cos^2\alpha = \frac{2}{100} \qquad / \cos\alpha < 0 \text{ because } \alpha \in (90°, 180°)$$

$$\cos\alpha = -\frac{\sqrt{2}}{10}$$

$$\sin\alpha = -7 \cdot \cos\alpha = \frac{7\sqrt{2}}{10}$$

Answer: $\cos\alpha = -\frac{\sqrt{2}}{10}$ $\sin\alpha = \frac{7\sqrt{2}}{10}$

Task 4. Calculate without calculator:

(a) $(2 + \cos150°)(2 - \sin120°) + \text{tg}135° = \left(2 - \frac{\sqrt{3}}{2}\right)\left(2 - \frac{\sqrt{3}}{2}\right) - 1 = 3\frac{3}{4} - 2\sqrt{3}$

(b) $\sin^2150° + \cos^2120° + \text{tg}^2135° = \sin^230° + \cos^260° + \text{tg}^245° = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 1^2 = 1\frac{1}{2}$

(c) $\tan20° \cdot \tan51° \cdot \tan30° \cdot \tan39° \cdot \tan70° = \tan20° \cdot \tan51° \cdot \frac{\sqrt{3}}{3} \cdot \cot51° \times \cot20° =$

$= \frac{\sqrt{3}}{3} \cdot \tan20° \cdot \cot20° \cdot \tan51° \cdot \cot51° = \frac{\sqrt{3}}{3} \cdot 1 \cdot 1 = \frac{\sqrt{3}}{3}$

Answers: (a) $3\frac{3}{4} - 2\sqrt{3}$ (b) $1\frac{1}{2}$ (c) $\frac{\sqrt{3}}{3}$