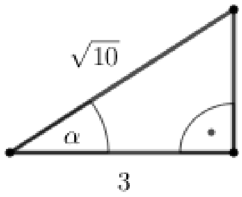
Task 1. Calculate value of the trigonometric expression below, using data given in the figure.

$(\tan\alpha + 2\cot\alpha)\cdot\sin\alpha\cdot\cos\alpha =$



Solution 1

$(\tan\alpha + 2\cot\alpha)\cdot\sin\alpha\cdot\cos\alpha = \tan\alpha\cdot\sin\alpha\cdot\cos\alpha + 2\cot\alpha\cdot\sin\alpha\cdot\cos\alpha =$

$= \frac{\sin\alpha}{\cos\alpha}\cdot\sin\alpha\cdot\cos\alpha + 2\cdot\frac{\cos\alpha}{\sin\alpha}\cdot\sin\alpha\cdot\cos\alpha = \sin^2\alpha + 2\cdot\cos^2\alpha =$

$= \sin^2\alpha + \cos^2\alpha + \cos^2\alpha = 1 + \cos^2\alpha = 1 + \left(\frac{3}{\sqrt{10}}\right)^2 = 1 + \frac{9}{10} = 1.9$

Solution 2

Let x be the length of the site opposite the angle α.

By Pythagoras's Theorem: $x^2 = \left(\sqrt{10}\right)^2 - 3^2 = 10 - 9 = 1.$

Thus $x = 1$ and $\sin\alpha = \frac{1}{\sqrt{10}}, \cos\alpha = \frac{3}{\sqrt{10}}, \tan\alpha = \frac{1}{3}, \cot\alpha = 3.$

Therefore $(\tan\alpha + 2\cot\alpha)\cdot\sin\alpha\cdot\cos\alpha = \left(\frac{1}{3} + 2\cdot 3\right)\cdot\frac{1}{\sqrt{10}}\cdot\frac{3}{\sqrt{10}} = \frac{19}{3}\cdot\frac{3}{10} = \frac{19}{10} = 1.9$

Answer: 1.9

Task 2. Calculate the expression below and write the answer in the simplest form.

$(\text{ctg}45° + \text{tg}60°)(\cos60° - \sin60°) =$

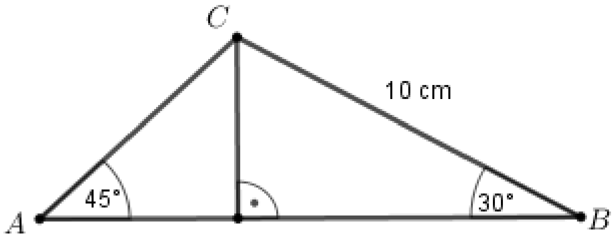
Solution

$(\text{ctg}45° + \text{tg}60°)(\cos60° - \sin60°) = \left(1 + \sqrt{3}\right)\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)$

$= \frac{1}{2}\cdot\left(1 + \sqrt{3}\right)\left(1 - \sqrt{3}\right) = \frac{1}{2}\cdot\left(1^2 - \left(\sqrt{3}\right)^2\right) = \frac{1}{2}\cdot(1 - 3) = -1$

Answer: $-\frac{1}{2}$

Task 3. Find the perimeter of the triangle ABC using data in the picture.



Solution (by using special right triangles)

$$\text{Perimeter}(ABC) = |AB| + |AC| + |BC| = (5\sqrt{3} + 5) + 5\sqrt{2} + 10 = 15 + 5\sqrt{2} + 5\sqrt{3}$$

Answer: $15 + 5\sqrt{2} + 5\sqrt{3}$

Task 4. An acute angle α satisfies the equation $\sin\alpha + \cos\alpha = \frac{6}{\sqrt{26}}$. Calculate values of the following expressions: (a) $\sin\alpha \cdot \cos\alpha$, (b) $\sin^4\alpha + \cos^4\alpha$.

Solution a	**Solution b**
$\sin\alpha + \cos\alpha = \frac{6}{\sqrt{26}}$ $(\sin\alpha + \cos\alpha)^2 = \frac{36}{26}$ $\sin^2\alpha + 2\cdot\sin\alpha\cdot\cos\alpha + \cos^2\alpha = \frac{18}{13}$ $\sin^2\alpha + + \cos^2\alpha + 2\cdot\sin\alpha\cdot\cos\alpha = \frac{18}{13}$ $1 + 2\cdot\sin\alpha\cdot\cos\alpha = \frac{18}{13}$ $2\cdot\sin\alpha\cdot\cos\alpha = \frac{5}{13}$ $\sin\alpha\cdot\cos\alpha = \frac{5}{26}$	$\sin^4\alpha + \cos^4\alpha =$ $= (\sin^2\alpha + \cos^2\alpha)^2 - 2\cdot\sin^2\alpha\cdot\cos^2\alpha =$ $= (\sin^2\alpha + \cos^2\alpha)^2 - 2\cdot(\sin\alpha\cdot\cos\alpha)^2 =$ $= 1^2 - 2\cdot\left(\frac{5}{26}\right)^2 = 1 - 2\cdot\frac{25}{676} =$ $= \frac{626}{676} = \frac{313}{338}$

Answer: (a) $\frac{5}{26}$ (b) $\frac{313}{338}$