

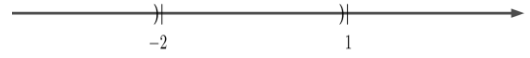
Inequality with two absolute value expressions

Task. Solve the inequality the $|x - 1| - 6|x + 2| < -2$

Solution

The expression $x - 1$ is equal to 0 for $x = 1$.

The expression $x + 2$ is equal to 0 for $x = -2$.



So we divide the solution into 3 cases

Case 1: $x \in (-\infty, -2)$

In this case $x + 2 < 0$ and $x - 1 < 0$, so $|x + 2| = (-x - 2)$ and $|x - 1| = (-x + 1)$.

Case 2: $x \in [-2, 1)$

In this case $x + 2 \geq 0$ and $x - 1 < 0$, so $|x + 2| = (x + 2)$ and $|x - 1| = (-x + 1)$.

Case 3: $x \in [1, \infty)$

In this case $x + 2 \geq 0$ and $x - 1 \geq 0$, so $|x + 2| = (x + 2)$ and $|x - 1| = (x - 1)$.

Case 1		Case 2		Case 3
$x \in (-\infty, -2)$		$x \in [-2, 1)$		$x \in [1, \infty)$
$(-x + 1) - 6(-x - 2) < -2$		$(-x + 1) - 6(x + 2) < -2$		$(x - 1) - 6(x + 2) \leq -2$
$5x < -15$		$-7x < 9$		$-5x < 11$
$x < -3$		$x > -\frac{9}{7}$		$x > -\frac{11}{5}$
$x \in (-\infty, -3)$		$x \in \left(-1\frac{2}{7}, \infty\right)$		$x \in \left(-2\frac{1}{5}, \infty\right)$
$x \in (-\infty, -3) \cap (-\infty, -2)$		$x \in \left(-1\frac{2}{7}, \infty\right) \cap [-2, 1)$		$x \in \left(-2\frac{1}{5}, \infty\right) \cap [1, \infty)$
$x \in (-\infty, -3)$	or	$x \in \left(-1\frac{2}{7}, 1\right)$	or	$x \in [1, \infty)$

Sum up the obtained sets of solutions $x \in (-\infty, -3) \cup \left(-1\frac{2}{7}, 1\right) \cup [1, \infty)$ and simplify the result.

Answer: $x \in (-\infty, -3) \cup \left(-1\frac{2}{7}, \infty\right)$