Inequality with two absolute value expressions

Task. Solve the inequality the $|x-1|-6|x+2|<-2$

Solution

The expression $x-1$ is equal to 0 for $x=1$.

The expression $x+2$ is equal to 0 for $x=-2$.

So we divide the solution into 3 cases

Case 1: $x\in(-\infty,-2)$

In this case $x+2<0$ and $x-1<0$, so $|x+2|=(-x-2)$ and $|x-1|=(-x+1)$.

Case 2: $x\in[-2,1)$

In this case $x+2\geq 0$ and $x-1<0$, so $|x+2|=(x+2)$ and $|x-1|=(-x+1)$.

Case 3: $x\in[1,\infty)$

In this case $x+2\geq 0$ oraz $x-1\geq 0$, so$|x+2|=(x+2)$ and $|x-1|=(x-1)$.

Case 1		Case 2		Case 3
$x\in(-\infty,-2)$		$x\in[-2,1)$		$x\in[1,\infty)$
$(-x+1)-6(-x-2)<-2$		$(-x+1)-6(x+2)<-2$		$(x-1)-6(x+2)\leq -2$
$5x<-15$		$-7x<9$		$-5x<11$
$x<-3$		$x>-\frac{9}{7}$		$x>-\frac{11}{5}$
$x\in(-\infty,-3)$		$x\in\left(-1\frac{2}{7},\infty\right)$		$x\in\left(-2\frac{1}{5},\infty\right)$
$x\in(-\infty,-3)\cap(-\infty,-2)$		$x\in\left(-1\frac{2}{7},\infty\right)\cap[-2,1)$		$x\in\left(-2\frac{1}{5},\infty\right)\cap[1,\infty)$
$x\in(-\infty,-3)$	or	$x\in\left(-1\frac{2}{7},1\right)$	or	$x\in[1,\infty)$

Sum up the obtained sets of solutions $x\in(-\infty,-3)\cup\left(-1\frac{2}{7},1\right)\cup[1,\infty)$ and simplify the result.

Answer: $x\in(-\infty,-3)\cup\left(-1\frac{2}{7},\infty\right)$