

Absolute and relative approximation error.

Exercise 1. Display the number $\sqrt{2020}$ on the calculator and give its approximation to the nearest:

- (a) tens **40** (b) tenths **44.9**
 (c) hundredths **44.94** (d) thousandths **44.944**
 (e) four decimal places **44.9444** (f) units **45**

Calculator display: $\sqrt{2020} = 44.94441010\dots$

Exercise 2. Use a **calculator** to provide approximate values of the given expressions with an accuracy of two decimal place. Exercise (b) is done for you.

(a) $4\sqrt{17} - 3 = 13.492\dots \approx 13.49$

17	$\sqrt{}$	\times	4	=	M+	3	=	M-	MRC
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(b) $7\sqrt{23} - 3\sqrt{5} = 26.86261673 \dots \approx 26.86$

To get the result on your simple calculator you should press the following keys in order:

23	$\sqrt{}$	\times	7	=	M+	5	$\sqrt{}$	\times	3	=	M-	MRC
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(c) $\frac{7\sqrt{15} - 5\sqrt{10}}{10} = 1.129\dots \approx 1.12$

15	$\sqrt{}$	\times	0.7	=	M+	10	$\sqrt{}$	\times	0.5	=	M-	MRC
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Exercise 4. Check, without using the calculator, whether the given number belongs to the P interval.

Hint: Use perfect squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, ...

(a) $\frac{8+3\sqrt{75}}{2}$ $P = (17\frac{1}{2}, 18)$ (b) $\frac{27-\sqrt{180}}{5}$ $P = (2\frac{1}{2}, 3)$

Solution (a)	Solution (b)
$\frac{8+3\sqrt{64}}{2} < \frac{8+3\sqrt{75}}{2} < \frac{8+3\sqrt{81}}{2}$ $16 < \frac{8+3\sqrt{75}}{2} < 17\frac{1}{2}$ $\frac{8+3\sqrt{75}}{2} \notin (17\frac{1}{2}, 18)$	$\frac{27-\sqrt{196}}{5} < \frac{27-\sqrt{180}}{5} < \frac{27-\sqrt{169}}{5}$ $2.6 < \frac{27-\sqrt{180}}{5} < 2.8$ $\frac{27-\sqrt{180}}{5} \in (2\frac{1}{2}, 3)$
No	Yes

Exercise 5. You are given a cube of pure copper. You measure the sides of the cube to find the volume and weigh it to find its mass. When you calculate the density using your measurements, you get 8.78 grams/cm³. Copper's accepted density is 8.96 g/cm³. What is your percent error?

Solution

$$\text{Percent error} = \frac{|8.78-8.96|}{|8.96|} \times 100\% = \frac{0.18}{8.96} \times 100\% = 0,11160 \dots \approx 11\%$$

Exercise 6. The approximation by excess of the number x is equal to 13.8, and the absolute error of this approximation is 0.12. Find the number x .

Solution

$$x \approx 13.48 \quad \text{and} \quad x < 13.8$$

$$13.8 - x = 0.12$$

$$x = 13.68$$

Exercise 7. The approximation by defect of x is 10.4 and the error is the relative approximation is 0.13. Calculate x .

Solution

$$x \approx 10.4 \quad \text{and} \quad x > 10.4$$

$$\frac{x-10.4}{x} = 0.13$$

$$x - 10.4 = 0.13x$$

$$0.87x = 10.4$$

$$x = \frac{10.4}{0.87}$$

$$x = 11 \frac{83}{87}$$

English	Polish
approximation	przybliżenie
decimal approximation	przybliżenie dziesiętne
approximation to tens	przybliżenie z dokładnością do dziesięciu
approximation to tenths	przybliżenie z dokładnością do jednej dziesiątej
approximation to four decimal places.	przybliżenie z dokładnością do 4 miejsc po przecinku
with an accuracy of two decimal place	z dokładnością do dwóch miejsc po przecinku
approximation by excess	przybliżenie z nadmiarem
approximation by defect	przybliżenie z niedomiarem
perfect square	kwadrat zupełny (kwadrat liczby naturalnej)